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SOME DIFFICULTIES IN TEACHER – STUDENT RELATIONS IN THE CONTEXT OF PROBLEM SOLVING.

Using mathematical tools to describe introduced in particular problem situations makes basic ability that allows an individual student to develop mathematical skills. Proper analysis of the problem, thinking about the way of solution, accomplishing every step that was planned before and finally looking back on the whole problem from another perspective provide the foundation of the reasoning that leads to abstract thinking about considering text. George Polya's method of solving mathematical problems, which had to be modify due to constant digitalization of the reality, is still very important in the teacher – student relations.
In the article I will show the method of solving some particular problems from Polish national exams, which every teacher should go with his students to achieve desired educational effects. I will also focus on some difficulties that can appear during the process of problem solving in the relation between teacher and students. That difficulties could not only become a reason of students' failure in mathematical learning but also could create bad habits in teachers' attitude.

Keywords: didactics of mathematics, problem solving

Interpersonal skills nowadays are mainly reduced to virtual communication. For many years people have been creating digitalized reality. Teachers have been complaining about their students' weak ability of reading and understanding what they have read. Young people are used to short text massages, casual information and mental shortcuts. They are addicted to constant emotional signs delivered by mobile transmitters, thus young people cannot focus on traditional, printed text – long, changeless, often monotonous. The whole digitalization process though started many years ago and despite its steady escalation, current "young adults" are already digital generation and written, printed text is much less close to them than messages displayed on their smartphones or notebooks. These "young adults" are, among others, teachers whose obligation is to equip their pupils with the basics of the broad sense of mathematical knowledge. School education is strictly connected with a verbal communication during math classes, with the clear and complete transmission and in consequence – with using correct, precise mathematical language. It is necessary to oberve communication process during math classes performing by these young teachers and also for the forms of information transfer among students of teacher's field of study. The main teachers' role and obligation is to give their students abilities of proper using and applying mathematical tools to describe situations introduced in particular problems.

Proper analysis of the problem, thinking about the way of solution, accomplishing every step that was planned before and finally looking back on the whole problem from another perspective provide the foundation of the reasoning that leads to abstract thinking about considering text. Problem solving or thinking about the proof of particular statement requires energy, devotion and concentration. In problem solving or proving some statements teachers may find some opportunities to create curious, open-minded young discoverers. It is not an easy job to do though, because there is a great risk of killing pupils' enthusiasm by teacher's skepticism, there is a large chance to nip pupils' energy in the bud by routine operations and there is a huge possibility to discourage pupils' endeavors by giving them wrong - chosen problems to solve or using inappropriate forms or methods of their work.

PRINCIPLES OF PROBLEM SOLVING

George Polya once said that "the aim of mathematics in the high school curriculum should be to teach young people to think" (Polya, 1965, p.100) and he identified such thinking with problem solving. "For mathematics education and for the world of problem solving [Polya's work] marked a line of demarcation between two eras, problem solving before and after Polya" (Schoenfeld, 1987, p.27). Polish educational system is facing serious changes both in its structure and its contents. Reasoning and mathematical thinking would be much more emphasized, thus methods of problem solving and techniques that help how to teach it seems to be necessary tools in every teacher's workshop.

Polya's first principle: Understand the problem, is so obvious that very often it is not even mentioned by the teacher during analysis of the considering problem. Yet, students failures are sometimes the result of not fully understanding the task they had to accomplish or even not understanding it in part. In the book *How to solve it* Polya suggests series of questions that every teacher could use during this first phase of problem solving:

Do you understand all the words used in stating the problem? What are you asked to find or show? Can you restate the problem in your own words? Can you think of a picture or diagram that might help you understand the problem? Is there enough information to enable you to find a solution? (Polya, 1945)

The dialoque between teacher and students should lead to the second principle: Device the plan. Almost every problem has several ways to be solved and the ability at choosing an appropriate strategy is crucial and the teacher should discuss the whole strategy with pupils, so they could proceed on their own if they needed to. The only possibility to get the skill of creating a strategy is by solving many problems. We can notice some analogy or use some deductive methods when we have met similar problem before, we are able to use our experience when we achieved similar conclusions before.

Carrying out the plan, the third step in problem solving, is usually easier than previous ones. When students once understood their needs in particular problem and planned its solution, the teacher can count on their abilities of doing what they have planned before. Students have to persist with the plan they have chosen and if it fails, they have to choose another one, because that is exactly how problem solving is being done.

The last principle in Polya's problem solving is looking back at what we have done. This step is very often ignored also by students and their teachers. Polya suggests that a lot can be gained by taking the time to reflect, to notice what worked and what did not. Doing it will help to predict what strategy use in solving forthcoming problems.

It is well known statement that mathematics is very difficult subject in school education and the reason for that common reflection is probably hidden behind the abstract meaning of the mathematical conceptions and its deductive structure. Problem solving was the subject of interest of many didactics of mathematics, but mostly, their interest was focused on children, primary school pupils (i.a. Ciosek 1978, Legutko 1989, Siwek 1984, Treliński 1984). Methods and strategies of problem solving for the older pupils should remain the same, they should have had "good habits" already but the high school reality shows that it is not a true assumption. There are many reasons of difficulties coming from problem solving, such as students' weak ability of reading, understanding and analyzing the problem. Pupils cannot use former knowledge and they are not able to notice any analogy. Sometimes their factual mathematical knowledge is not sufficient to solve the problem, but very often they just cannot choose the right method for the problem and think about the right way to go.

Each mathematical problem consists of two stages and requires different activities during solving it. The first stage is verbal text that represents some empiric situation (real or imaginary) that the reader needs to understand and analyze properly. This leads straight to the second stage, that is mathematic layer of the problem. It represents abstract activities depending on the right transformation of the information taken from verbal text into mathematical equations to solve. Problem solving demands forming mathematical statements with their strict language and unambiguous meaning. Using proper terminology demands noticing every necessary detail, thus transfer from the first stage to the second is very difficult and not every student is prepared to do so. Methodology of problem solving needs to be use very careful and systematically. Solving problems that requires two mentioned stages is a great way of learning mathematics because it leads to deep understanding of the knowledge we have learnt before and new abilities. Furthermore, it helps to understand some methods of reasoning, the sense of generalization, noticing and using analogy and much more.

CHARACTERIZATION OF CASE-STUDY EXPERIMENT

In the following article there is some concept introduced, the concept about checking if the methodology of problem solving is being understood correctly by students of mathematics who are trained to be teachers . Conclusions that will be make in the paper are not generalized, there are only results of some case – study example, but they can determine wider picture of the analysis of the problem solving in teaching process. The particular case – study was about getting to know if students understood what they were taught during classes, so it was some kind of observation of the didactic process. The experiment involved eight students of mathematics, 2nd year , 1st degree studies (bachelor's degree). During Basic of Didactics classes students were familiarized with Polya's methods of problem solving in details, there were a lot of examples of using that methods discussed. Students awareness were raised to notice the importance and meaning of asking right questions and the role of deep analysis of the considering problem, before solving it, that is before making every step from the plan that was prepared before. Furthermore, students have already had some experience with teaching, working with pupils, performing a lesson of mathematics, because simultaneously they were attending practice classes in primary school as obligatory course classes – they performed lessons in the practice-school and their lessons were analyzed very deeply taking care of its basis of contents, stylistic correctness and educational value.

The experiment consisted of solving given problems by students, preparing a script of solution and perform a "lesson" in front of the rest of the group like in classroom full of pupils. Their task was to work with "pupils" including Polya's methodology of problem solving. Each student were given two problems and they all had enough time to solve them and prepare a plan to proceed. Everyone knew only their own tasks, they did not know what their colleagues have got.

The concept that was described above, was supposed to be some kind of monitoring of the assimilation of the classes contents but the results of that monitoring and observations that was done during the experiment became so interesting and surprising that they appoint a direction of research that could be done on a wider scale. Conclusions that could be done after mentioned research could be essential in the context of problem solving teaching.

PRESENTATION OF THE PROBLEMS

In the following paper there will be two problems introduced and discussed in details. Based on these analysis there will be made some conclusions and some indication to more detailed research. The particular problems appeared on Polish national exam that pupils pass after secondary school (it is called "matura") in 2018, basic and advanced level. Both problems come from the 5th standard of general requirements contained in current decree of the National Minister of Education about general education in various types of schools, that is – Reasoning and argumentation. From the pupil on basic level an ability of performing easy reasoning containing a small number of steps is demanded. Necessary knowledge that a pupil needs to have in considered in the following article problem is reduced to awareness of short multiplications formulas and basic ability of transforming rational expressions. On advanced level a pupil needs to be able to create a chain of arguments and explaining its correctness. The particular problem introduced in the paper is about divisibility of numbers and it requires basic characteristics of real numbers and an ability of polynomials' decomposition into factors.

The problems present as follows:

Problem 1 (Problem 28, CKE, May 2018, basic level)
Prove that for any positive numbers *a, b* the following inequality is true:

$$\frac{1}{2a}+\frac{1}{2b}\geq\frac{2}{a+b}.$$

An exemplary solution of the problem:
Since *a, b* are positive numbers, then $2a>0,\ 2b>0,\ a+b>0$. Let's multiply both sides of an inequality by $2a,\ 2b,\ a+b$. We have got:

$$2b(a+b)+2a(a+b)\geq 2\cdot 2a\cdot 2b$$

$$2ab+2b^2+2a^2+2ab-8ab\geq 0$$

$$2a^2-4ab+2b^2\geq 0$$

$$2(a-b)^2\geq 0.$$

The inequality above is true for any numbers, so in particular, for any positive numbers. It ends the prof.

Problem 2 (Problem 8, CKE, May 2018, advanced level)
Prove that for any integer *k* and for any integer *m*, the number k^3m-km^3 is divided by 6.

An exemplary solution of the problem:
Method I: Let's notice that:

$$k^3m - km^3 = km\left(k^2 - m^2\right)$$
$$= km\left(k - m\right)\left(k + m\right).$$

The solution consists of two stages: explaining divisibility by 2 and divisibility by 3.

Divisibility by 2: If any of numbers *k, m* is even, then *km* is even, so the whole expression $km\left(k^2 - m^2\right)$ is even. The case both numbers are odd, the sum *k* + *m* is even, so the product $km\left(k - m\right)\left(k + m\right)$ is divided by 2.

Divisibility by 3: Let's consider four cases:
1. One of the numbers *k, m* is divided by 3. Then the product *km* is divided by 3, so the whole expression $km\left(k^2 - m^2\right)$ is also divided by 3.
2. Both numbers *k, m* in division by 3 give the remainder of 1. Then $\left(k - m\right)$ is divided by 3, so is the product $km\left(k - m\right)\left(k + m\right)$.
3. Both numbers *k, m* in division by 3 give the remainder of 2. Then $\left(k - m\right)$ is divided by 3, so is the product $km\left(k - m\right)\left(k + m\right)$.
4. One of the numbers *k, m* in division by 3 gives the remainder of 1 and the other number in division by 3 gives the remainder of 2. Then $\left(k + m\right)$ is divided by 3, so the product $km\left(k - m\right)\left(k + m\right)$ is divided by 3.

We have shown that the number $k^3m - km^3$ is divided by 2 and by 3, so it is divided by 6. It ends the prof.

II method: Let's notice that

$$k^3m - km^3 = km\left(k^2 - 1 + 1 - m^2\right)$$
$$= km\left(k^2 - 1\right) - km\left(m^2 - 1\right)$$
$$= km\left(k - 1\right)\left(k + 1\right) - km\left(m - 1\right)\left(m + 1\right).$$

Since the expression $k\left(k - 1\right)\left(k + 1\right)$ is a product of three succeeding numbers, then one of them must be divided by 3 and at least one of them is divided by 2. Thus the whole product is divided by 6. Annalogically, the product $m\left(m - 1\right)\left(m + 1\right)$ is divided by 6. The difference of two numbers that are divided by 6 is also divided by 6. It ends the prof.

RESULTS OF THE EXPERIMENT

Students, whose task was to perform these particular problems in front of the whole group, working with the group to achieve the solution by Polya's method of problem solving, were fully prepared for the job. They had a script of several solutions, helpful questions and potential pupils' ideas.

1. Results and analysis of the teacher's work during solving Problem 1 in the classroom

Student's first action in the classroom was starting a short discussion about the assumptions and the thesis of considering problem. Arranging a plan of solution was reduced to the statement, that series of transformations of the given expression needs to be done to obtain some identity inequality. The future teacher led the main thought of the group, he appointed the direction of the discussion and he took care of its correctness. Only during executing prepared plan, specifically during making particular transformations of the rational expression appeared some disturbing signals about unreliableness of the method of student's work. It was noticeable that pupils did not understand the sense of the prof correctly. Starting with the thesis of the problem, that is an inequality $\frac{1}{2a} + \frac{1}{2b} \geq \frac{2}{a+b}$, teacher asked:

– *What is our first transformation going to be?* The pupil answered: – *We can bring left side to the common denominator.* The other pupil added: – *Let's bring everything to the common denominator at once, it would be easier (…)*

Teacher should sum up arisen discussion with the comment about the lack of the easiest transformation, about individual preferences and different ways of choices, but he said instead:

– *The easiest way would be to get rid of all denominators at once. How can we do that?*

Student did not give his pupils any opportunity to choose their own way, he did not allow them to think about their own ideas and he did not even admit their correct and proper suggestions. Such situations should not take place during any math classes but especially during performing a prof. Pupils are afraid of that kind of mathematical problems in general, they fear the possibility of the wrong choice, they fear the wrong ideas and they feel insecure because they are afraid of a failure. **The teacher that does not appreciate pupils' efforts, that does not encourage them to finding some rules and claims on their own, he only escalates their fear of a failure and he nips signs of any creativity in the bud.** Mentioned situation is additionally much worse because the teacher ignored pupils' suggestions although both of the ideas were correct. The student should react on suggested proposals, even if he wanted to continue with his own method. The comment like follows would be sufficient: - *We can bring the left side to the common denominator, we can also bring everything to the common denominator at once. Both ideas are good and they both lead to the same result. What is that?* It would provoke next discussion, more or less directed by the teacher, but very eye-opening and sustaining pupils' creative attitude. After that kind of discussion, teacher should let his pupils act like they have chosen and he should control their actions. After the classes, when student was telling the rest of the group about reasons of his behavior, he claimed that he assumed that everything was so clear and obvious that none comment was necessary. **The teacher that does not understand the role and importance of solving the problem with the method chosen on the result of someone's own ideas, is not able to develop pupils' responsive attitude and excite creative brains to more and more courageous and ambitious actions.** Teacher should even appreciate pupils' commitment into proving some statement too much sometimes just to let them think, that they discovered at least the part of solution by themselves.

Moreover, removing denominators (by making any kind of correct transformations), that is multiplying by denominators, we use assumptions given in the problem. Solving that kind of problems with pupils, teacher should pay special attention for moments when we use assumptions to make sure that pupils will understand the whole idea of proving properly, that they will acknowledge the need of explaining every step they take and will get the consciousness of doing everything correctly. The student mentioned admittedly that we multiply by positive expressions so we do not have to worry about the sign of inequality but the comment seemed to be very meaningless against performed actions.

When considered inequality was brought to the form of identity inequality $2(a-b)^2 \geq 0$, the teacher summarized the whole problem with one short comment:

– *We have already achieved inequality that is always satisfied by any number, so it ends our prof.*

The forth rule of Polya's method of problem solving was practically not applied at all and during the discussion after performing classes, the student confirmed that his actions was intended, that in "that kind of problem" there was nothing more to talk about, that looking back is an unnecessary step. It is very wrong belief and unfortunately it shows immaturity of the future teacher's mind. Pupils are inexperienced but really responsive and every problem, especially that one that involves proving some statement, should be discussed one more after performing a proof. That is a good moment to show various ways of reasoning, to compare deductive and reductive methods, to discuss necessity of analyzing and verifying every step, to pay attention to the order and correctness of the notation. **It is teacher's duty to imprint in pupils' mind the need of explaining and justifying their thoughts and to teach them the ways of arguing and reasoning during performing a proof.**

Discussion with the whole group of students after solving and talking through the particular problem was very illuminating and it caused a fresh look on the idea of proving in school education. Students are used to proving every mathematical statements, they treat it like something completely normal, natural and even mechanical. That is the reason they often forget that proving for school pupils is a whole new ability they need to learn and understand to be able to manage it.

2. Results and analysis of the teacher's work during solving Problem 2 in the classroom

Problem 2 is dedicated to pupils that learn mathematics on advanced level so teacher can expect more commitment from them, the discussion should be more factual and pupils' knowledge should be more complete. Teacher's language should always be adjusted to intellectual level of its receivers but because of a bigger amount of mathematical classes in education course and much more precise material, those pupils should be more interested in getting mathematical knowledge and the level of their mathematical knowledge should be higher than the level of mathematical knowledge among pupils that learn only basic mathematics. The student, who prepared for solving that problem with pupils, came up with two solutions of the problem that were described above. The future teacher admitted during the first discussion over her script, that it was really difficult for her to prepare for the job. She admitted that solving the problem she used boilerplate and that she felt very insecure about it. The first and obvious conclusion arising after the discussion is that students of mathematics (teaching specialty) solve not enough problems of school mathematics, problems they would have to deal with in the classroom full of pupils. After graduation of mathematical studies, young teachers are used to recreated knowledge, something they have to introduce, not necessarily explain. It matters at the beginning of their professional careers because when they find any obstacles during solving particular problem, they will search for already prepared solution instead of trying to solve the problem by themselves. They will not discover any mathematical activities they might go through on the way of finding the solution. Teachers that proceed like that will not be able to point "the right way" to their pupils, they will not be able to go through that way with them and most importantly they will not be able to teach them finding such ways.

Another disturbing signal that appeared during the discussion before performing a lesson by a student and it was student's question about solving the problem using the second method straight away. She deduced that writing down and explaining all the cases in the first method of solution was very time – consuming and the second method was just much easier to perform in the classroom. On the question: Did you start working on the solution of that problem just the way you want to proceed? Did the method described in the second solution was your first idea when you read the problem initially? Student's answer was quick: No, of course not, I started with the decomposition of the polynomial and then I tried to write down cases, like in the first method. Student was not aware of the didactic mistake she was about to make, she just saw apparent difficulty (confuse and time – consuming writing down cases) so she did not want to dig up to that difficulty with pupils. Such an attitude in the classroom is unacceptable. **If student started to bring over the solution of the problem only to the second method, the whole analysis of particular problem and all the way to achieve desired result would be artificial and it would cause the feeling of helplessness and mathematical vulnerability of failing**. Pupils would obviously understand the second method and the way of solution but the lack of experience in mathematical "tricks" and not stereotypical way of thinking would undermine their self–consciousness and self-belief or it could even discourage them from their further work. The pupils need to believe that they find a solution by themselves, even if it is given to them from outside. The teacher's duty is to navigate the pupils' thinking, precisely by asking right questions, that leave pupils with the feeling of mathematical fulfillment in considering field.

After deep analysis of the didactic value and a structure of considering problem, student understood how important and meaningful is going through the first method of solution. During classes she started to discuss the problem with the group with proper use of Polya's method of problem solving. She applied appropriate series of questions, took care of the right question gradation depending on the reactions of the group. All the stages of the work with group of pupils were deeply discussed before actual performing the problem in class. The first stage of problem solving (understanding the problem) student achieved desired effects by using short list of questions: *What the problem is about? What does it mean that a number is divided by 6? What is the first thing you consider looking at given expression?* The discussion led to factorization of the polynomial and building a plan of solution – to explain why the expression is divided by 2 and by 3:

– *How can we explain that our expression is divided by 2?*

– *We consider some cases. First, let m and n be even numbers, then let m be an even number and m be an odd number, then we examine the other way round and finally, let m and n be odd numbers.*

– *Fine, but in our solution it is enough to explain that any number m or n is even, or both m and n are odd (…)*

The comment above deprived pupils of very eye-opening analysis of considering problem, that is noticing the analogy in cases, where one of numbers is even and the other one is odd. Pupils would certainly notice the analogy by themselves during explaining each of mentioned cases and then they would experience very important mathematical activity – modification of their own idea, improving it, rationalizing it. Such activities might build strong mathematical values that develop pupil's interest in mathematics. The teacher should aim for situations like that and not avoid them on purpose because it requires more time.

After discussing two cases of divisibility by 2 that student implied, there were a whole new conversation between a student and a group about the way of explaining divisibility by 3. The teacher asked:

– *What cases do we need to consider to show that our expression is divided by 3?*

– *Two cases. One, that any number is divided by three, and the other one – that none of the numbers is divided by 3, just like before.*

The teacher answered:

– *But in this case there could be different remainders, we will have more cases (…)*

Pupils' comment indicates unambiguously that the first stage of solving the problem has had some serious defects, that pupils has not understood the main idea of the proof correctly. Giving up the whole process of writing down each case pupils suggested caused misunderstanding that would be hard to repair. The teacher used a shortcut that for her pupils was believable but artificial. It was not a proof step, it was an argument they had to believe in. When analogical problem appeared with divisibility by 3, pupils wanted to use a shortcut either, because it had worked before. If described situation happened in the classroom, the teacher should be able to make an immediate conclusion, she should understand appearing mistake, hold on to it and explain it to pupils very carefully. That is the only way to prevent creating bad habits that would be hard to eliminate later.

CONCLUSIONS

The teacher who cannot analyze the mathematical problem correctly with pupils would not be able to develop creative attitude in them. Mathematical creativity is the most important activity on the way of abstract reasoning. Presenting our own point of view, verbalizing our own process of thinking, even if it is trivial, is the most effective way of learning. Teachers need to equip their pupils in that ability, so they have to teach them how to solve mathematical problems by themselves. Case study that was described in the following article is just a starting point to the series of research about solving mathematical problems from a teacher's point of view. Students that were involved in the experiment appreciated the value of it after performing their lessons, although initially they were very skeptical about it. After solving their tasks, preparing a script for their classes they even claimed that they knew exactly what to do and how and they did not feel any need to verify it. Planning every step of solution in a particular problem is necessary in teacher's work but inexperienced students do not assume that their own behaviors, their own comments and acts cause their pupils make mistakes. The group of students understood their own mistakes, they recognized the need of full, deep analysis of each problem and they finally appreciated the role and value of the form of communication and clear guidelines. They admitted that **it was very difficult job to explain a way of thinking to a group of pupils and to lead them through the way of mathematical reasoning**. They also claimed that the most important thing is to let pupils find their own way, keeping them on the right tracks without telling them what to do, be supportive but not intruding.

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PEWNE TRUDNOŚCI W RELACJACH MIĘDZY NAUCZYCIELEM A UCZNIEM W KONTEKŚCIE ROZWIĄZYWANIA ZADAŃ TEKSTOWYCH

ABSTRAKT

Operatywne stosowanie narzędzi matematycznych do opisu sytuacji przedstawionej w rozpatrywanym problemie jest kluczową umiejętnością pozwalającą na prawidłowy rozwój matematyczny ucznia. Prawidłowa analiza zadania, zrozumienie problemu, zastanowienie się nad sposobem rozwiązania, wykonanie zaplanowanych czynności i w końcu spojrzenie na zadanie w sposób całościowy, stanowią fundament rozumowania, które prowadzi do abstrakcyjnego pojmowania rozważanych treści. Polyowska metoda rozwiązywania zadań tekstowych, modyfikowana ze względu na cyfryzację rzeczywistości, jest w dalszym ciągu podstawą pracy nauczyciela z uczniem.

W publikacji skupię się na metodach rozwiązania wybranych zadań maturalnych, przedstawię drogę, którą nauczyciel powinien przejść ze swoimi uczniami podczas analizy rozpatrywanego problemu. Zwrócę również uwagę na trudności, jakie pojawiają się na tej drodze, na nieporozumienia, które stanowią istotną przeszkodę w komunikacji na lekcji, a także w ogólnej relacji nauczyciel – uczeń.

Słowa kluczowe: Proces nauczania matematyki, rozwiązywanie zadań tekstowych