MAKING QUESTIONS AS THE WAY FAVOURABLE FOR STUDENTS’ MATHEMATICAL ACTIVITY

“To question well means to teach well” is a quotation which has inspired creation of a successful method of teaching based on questions asked by the students in order to help them develop their skills and enhance their knowledge.

The questions as an inevitable part of school life do not need to appear only as a means of evaluating students’ progress. Neither do they need to be answered in one expected way or constitute a one-way form of communication.

The results so far lead to the conclusion that the students who work with the use of the “making good questions pattern” perform better. They find it significantly easier to find the solution of the tasks. They are also much more eager to face new challenge and find it extremely rewarding when they are successful. Students, who were under examination, claimed, that “making question method” made them to formulate a hypothesis, prove its correctness, question the ideas, sometimes withdraw previous conclusions.

In my article being focused on my recent developments I present the results of research conducted for my dissertation work.

Keywords: cognitivistics, mathematical education, levels of processing, new media

STUDENTS’ MATHEMATICAL ACTIVITY

The progress of civilization that has taken place over the last 20 years can’t result with no impact on students who sit in school benches nowadays. Even an issue of the evolution of mobile telephony proves this indisputably. Today’s students are bombarded with a variety of stimuli which has not been a case in the past. Dr. Dawid Wiener claims that today’s students are characterized by “Efficient absorption, poor processing. In other words, children quickly acquire knowledge, but they can’t make use out of it. Since they remember facts without consideration, they have a problem if there is a need to assess its weight, find a common denominator or indicate cause and effect relationships. (...) When there is a test of choice – they don’t have a problem. But when the questions are open – the notes are falling down.”¹

It is hard to disagree with him. As a teacher, I observe that students are accustomed to su-

¹ Wiener D., Jak przegrzewa się mózg, czyli Homo Sapiens na zakręcie, http://wyborcza.pl/magazine/1,124059,6925549,Jak_przegrzewa_sie_mozg_czyli_Homo_sapiens_na_zakrecie.html
perficial approach to information they receive. Everything they find on the Internet grabs their attention only for a moment. The rate of inflow of new data does not allow for their deeper analysis, reflection, consideration. Because also the content that appears does not require a reflection. It is light, easy and pleasant. Students’ reward center is constantly being stimulated, they are always hungry for the new dopamine release. Fluctuations on its level lead to disturbances in the learning process, for which it is important “to what extent a given stimulus allows to predict the reward”\(^2\). Anything that does not bring the expected reward almost immediately, which is not intelligible at once, which requires the dedication of time, attention and effort, is treated as unnecessary, pointless, unproductive and useless. Mark Prensky is of a similar opinion when he writes that today’s students have changed radically and they are different than those for whom the educational system has been designed\(^3\). Anyhow these changes are estimated, one can’t deny their appearance. Probably the brains of our students are different from the previous young generations. This is not about different number of neurons, different weight or other chemical processes. Brains of “digital natives” differ from the brains of “digital immigrants”\(^4\) in terms of neural connections. “Natives” differently receive and analyze data, preferring multitasking, graphical forms prefer over text. “The world has changed, the youth is changing, and the education remains as it used to be: it is focused at reconstruction, not creation. At memorizing, not at thinking. It burdens memory, floods with facts, instead of teaching evaluation and developing creative thinking, what has provided us with evolutionary success. In short, it aggravates the negative effects of excess stimuli “(Wiener). Satisfied or not we have to try to change the methods so as to “communicate with the language and style of our students. It does not mean we need change the content of what is important neither to negate everything that has been happening so far.”\(^5\)

The theory presented in 1972 by Fergus Craik and Robert S. Lockhart states that the chance to remember specific information grows when it has been transformed at a deeper, semantic level. This theory says that every information is processed by the same brain structures, the difference is in the depth of processing information (where depth is understood as the number and complexity of operations). Work efficiency as well as susceptibility to interference depends on the achieved level. The authors indicated three levels of processing:

1. Sensory data analysis – shallow, susceptible to interference, with very unstable processing results.
2. Semantic interpretation of the received signal – deep, including understanding the meaning of a given signal.
3. Third level – the deepest one, when the knowledge already possessed is been activating, reorganizing and supplementing and its new structures are created.

The conclusions from the works of Craik and Lockhart\(^6\) say that we will get much better results if the number of repetitions (otherwise


also effective, while talking about remembering) we replace with activity resulting with deepening the processing by arousal of associations and applying the self-reference effect.

Let’s look at few theses posed by Żylińska:

1. Effectiveness of teaching is a resultant of: student motivation, time devoted to the problem, depth of information processing.7
2. The quality of educational materials, the type of student activity and tasks prepared by the teacher are the basis of the learning process.8
3. The atmosphere in the classroom and good student-teacher relations are important factors influencing the effectiveness of teaching.
4. Open problems, allowing the use of knowledge and skills, making mistakes, putting hypotheses, give the student a chance for deep processing.9
5. Open questions bringing students into condition enabling to learn.10
6. Care should be taken about the teaching process, not just about its final result. 11
7. Surprising elements are immediately placed higher on the list of priorities.12

According to Żylińska, “solving typical tasks in workbooks does not require deep in-
formation processing, so it does not lead to memorizing information. Teachers should use the receptive and reproductive tasks less frequently, since they are rather measuring tools than exercises, and more often open and productive. The best results are achieved when the pupils create tasks themselves.”13

In one of his works, Klakla 14 writes about the Creative Activity of Students, having in mind the following aspects:

1. Making hypotheses and verifying them.
2. Transfer of the method.
3. Creative reception, processing and use of mathematical information.
4. Discipline and critical thinking.
5. Generating problems in the process of transfer method.
7. Putting problems in an open situation.

As the main features of creativity are proposed:

1. Transformation of phenomena, things, processes of actions or their images, views or sensory ones.
2. Novelty, originality of activity production, patterns or tools and means used in this activity.
3. Search for “unknown and existing links “ between the objects under consideration.15

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Also A. Z. Krygowska points out that “one should concentrate on these elements [of mathematical activity of students] and:

1. Study and analyze their functioning in creative mathematical work
2. Search for means to provoke them and develop at different levels of education
3. Examine favorable and unfavorable conditions for this development, conditions determined both by the content and structure of the program as well as didactic activity of the teacher.¹⁶

What is important, Klakla following W. A. Gusjew divides students into three categories¹⁷:

1. Students for whom mathematics is only an element of their general development
2. Students for whom mathematics will become an instrument in their professional activity
3. Students for whom mathematics will be the basis of their future activity.

QUESTIONS

Following Pobojewska’s work¹⁸, usually the function of the question is limited to checking the knowledge already possessed. The respondent is to give the expected answer. It is therefore necessary to break the usual conventions and build the relationship that will become the beginning of a “new question”. The catalog of “testing “ questions should be extended with a group of “genuine” questions. Let the pupil understand that there is something that he does not know, let him feel some uncertainty, let him stir up the “intellectual vigilance”, should he let himself for discomfort connected with the state of ignorance. This is one of the most significant moments. This momentum need to be noticed by the teacher, he should stimulate him and create a space where the student will be able to ask a question. Asking questions requires courage. You have to overcome the reluctance associated with shyness, shame, fear of speaking in front of the group, assessment of both the teacher and other team members. It is important in this context to create an atmosphere of mutual respect, understanding and cooperation considered as a joint effort aimed at solving the problem. It is necessary that the unnatural relationship between the examiner and the examined – shall be transferred into the relation typical for social relations, when we simply ask questions if we don’t know while desire to know. Building appropriate relationships is the starting point here. It is also needed to change the viewpoint of questioning person. So called “intellectual courage” – according to Pobojewska – must be popularized. Shall us remember that this process is not only about what the questioner does not know. After all, to consider it he must analyze and then synthesize his knowledge. In order to a question to arise in the student’s mind, he must name what he already knows, what he does not know and what he wants to know. These efforts are a great exercise for the brain, especially in the context of understanding ‘learning’.

It has been already noted by Skurzyński who stated that “it turns out that [the stu-
dent] has no starting points. Young people build new information almost in a vacuum.”

Therefore, suggesting how important it is to prepare a “foundation” for development of the new knowledge. The creation of “horizon of thought” (Smirnow, 1951) is to be the work based on making questions. Which, in turn, leads to a method that can be applied during lessons in order to increase pupils’ mathematical activity and a better understanding of the issues discussed.

WORK ON MATH’S LESSONS

The tool developed by Dan Rothstein and Luz Santana “Question Formulation Technique, QTF” has been used and its principles are described below. At the beginning there was a task to run the reasoning associated with the triangulation of any convex polygon. This topic was chosen due to the several important criteria:

1. Clear and clearly formulated, understandable to the respondents without additional explanations.
2. Focusing on one, clear issue.
3. Simplicity of the topic, giving the opportunity to solve regardless of the mathematical knowledge possessed.
4. The subject of interest, provoking to think, arousing the desire to find a solution.
5. New, fresh problem, most likely not discussed at school and unknown to the respondents.

The work was carried out in the following phases:

1. The respondents were asked to solve the problem. At the same time, their task was to create as many questions as possible in connection with the work on the issue. The respondents were not suggested how questions should look like and were not helped in formulating them. The only suggestions concerned the method of finding them (saving those which occurred naturally, creating a list of helping questions that give the direction, changing the affirmative sentences that appear in the course of work into questions).
2. The respondents were to qualify the questions they posed to two groups: closed and open questions. Then their task was to reformulate the questions open to closed and closed to open. In addition, they were asked for providing several advantages and disadvantages of each type of question.
3. The respondents were asked to prioritize questions. Their task was to indicate the most important questions – in their opinion – the most helpful or even essential on the way to solve the problem (in the case the problem has been solved) or – in the case of failure – to indicate those questions which answers would open the way to its solution.

Thirteen first-year students in the field of computer science participated in the study. They worked independently and they had sixty minutes to do the exercise. At the beginning, they were introduced to two definitions:

Convex set – a subset of a certain space containing, together with any two of its points, the segment connecting them. The space may be e.g. Euclidean, affine or linear (i.e. vector); in all cases, the scalar body is required to be ordered, usually the body of real numbers.

Triangulation is the division of a geometric figure into simplexes (triangles or tetrahedrons) in such a way that the common part of

20 Make Just One Change: Teach Students to Ask Their Own Questions, 2011 Harvard Education Press.
any two different simplexes is their common wall, common vertex, common side or common triangle or empty set. It is also required that simplexes that form triangulation, any restricted area should cross only a finite number. One can triangulate each polygon and each polyhedron.

These definitions were not commented or further clarified. Students were also not asked if they had known or understood them. Due to the fact that they were directed to students of engineering studies, the definitions have been presented in a strictly mathematical form. In principle, however, it is about simple things. The figure is called convex, if you choose two any points belonging to this figure and combining them with an episode, it will be completely contained in this figure, i.e. all points belonging to it will also belong to the figure. To better illustrate this, you can imagine a Christmas star figure. It is not a convex figure. By connecting two vertices with the segment, only its ends will belong to the figure. All other points of this episode will be outside of it.

It could be assumed, however, that the definition of a convex set appeared in a secondary school, while the definition of triangulation was new to them. This assumption was confirmed in later conversations with the participants. However, it is not difficult to understand. It’s about division of a given area (here a polygon) into triangles. It’s as if you choose one of the vertices in the hexagon and connect it with diagonals with the other three vertices (with two neighboring sides connected to it). In this way we get four triangles that form the starting hexagon.

Then the participants were asked to solve the following tasks:

**Exercise 1.**
Solve the task: How many different ways can you divide the \( n \)-angle convex into triangles with diagonals which don't cross each other?

a) Determine what it means that the two divisions are different.

b) How many diagonals do you need to divide the \( n \)-angle?

![Fig. 1. Example of a concave figure and a convex figure](#)
c) How many triangles are formed?
d) Solve the task for \( n = 3, 4, 5, 6 \).

**Exercise 2.**
Arrange a set of questions that could be asked (or which you have asked) while solving the exercise 1. Ask as many questions as possible. Record all of them. Replace the affirmative sentences with questions.

**Exercise 3.**
Turn your open questions into closed ones (and vice versa). Try to find a few advantages and disadvantages of both types of questions.

**Exercise 4.**
Rank out the questions. Try to arrange them from the most important to the least important in your opinion.

The problem was selected with the aim so that each participant, regardless of their previous experience and knowledge, could solve it. There are simple and intuitive operation and concepts to apply while solving this problem which makes it universal. The discussed problem was successfully presented to the respondents at three educational stages: third-grade grammar school students, first-grade high school students and first-year university students. This will allow to compare the results of study groups at various educational levels in future research. Their work on the subject was aimed at sending a series of stimuli to the respondents. The idea was to familiarize yourself with the vocabulary associated with the problem. It is important to note that it only was supposed to make introduction to the topic and was not related with the main task of dividing the polygon into triangles.

The relationship between the number and quality of created questions with the correctness and form of the solution has been evaluated. The analysis of the results obtained consisted in comparing two parts of the work: concerning questions and concerning solving a mathematical problem.

Analyzing the questions created by the respondents, the attention was primarily drawn to their substantive value. It was assessed whether they were related to the problem and connected with its solution. One of the indicators was also the number of questions created. Solutions have been divided into three groups: fully satisfying; correct, but having flaws and wrong solutions. Next, the questions formed by the respondents were evaluated within each group. There are the questions from the first group:

- In how many different ways can you divide the \( n \)-angle with diagonals which don’t cross each other?
- Is it possible to draw diagonals so that they do not cross, other than just from one vertex?
- Does drawing a diagonal from different vertices (for each “way” separately) give the same result or different?
- What is the relationship between diagonals and the number of triangles they share?

![Fig. 2. Example of triangulation of a hexagon](image-url)
• Are the triangles inside the diagonals counted as well?
• Is the division different from the elements or their number?

The above questions have been chosen because they appear (sometimes in a similar, though equivalent form) in each of the works, where the mathematical problem was correctly solved. There were five correct solved works. In each of them the number of questions ranged between 8 and 10. Some of them could be developed into independent issues.

It is worth noting that in the second group, where the solutions were wrong or there were none at all, the respondents were not able to create any questions or the recorded questions have not related anyway to the essence of the problem. The number of posed questions does not exceed five. Usually, they formulated them like following:

• How to verify it?
• Is it possible to verify it?
• Is there always one tetrahedron formed?
• Do it with graphic method?
• What does it mean to “solve the task for $n = 3, 4, 5, 6$”?

SUMMARY.

The aim of the article is to demonstrate practical application of a tool allowing students to become mathematically active and encourage them to work. In addition, it will help to develop and use the ability of asking questions and activation of the knowledge already possessed, provoking its reorganization, requiring the use of different types of associations. An important assumption is that it can be used both in working with a gifted student and those having problems with mathematics, regardless of which group among the groups – invoked by Klakla – the student is qualified. An important aspect is the implementation of the assumed didactic goals and achieving the intended results with reference to the internal motivation of students, with the respect for their creativity, autonomy and innovation. This method can be used during lessons in order to increase pupils’ mathematical activity and for better understanding the issues discussed. The essence is the pupil's work on the issue, extended in relation to the finding of the solution, additional activities related to the creation of a series of questions and their elaboration, allowing to increase the pupil's mathematical activity. The goal is to cause some preparatory activities that precede the task-solving process itself, allowing it to be placed as a natural implication of previous actions. This, in turn, will translate into a better understanding and longer retention of acquired knowledge in the time perspective.

Based on previous observations, the method of creating questions seems to be conducive to increasing the mathematical activity of students. Those who had showed greater involvement and commitment in creating and developing questions finally solved the problem correctly. In the interviews conducted after the end of the experiment, respondents admitted that the exercise based on creating a series of questions and then working with them, allowed them to return to the issue, rethink the method to solve it, consider the optimization of this process. A remark noted by one of the participants saying than creating and working on questions, including finding answers to own questions, may result with the solution of the main problem, clearly proves that the proposed form is effective. It is worth to emphasize that respondents weren't aware about the assumptions of the experiment and the hypotheses that should be verified with. In the presented study the phase of creation of questions has not been controlled in any way. The results suggest that the phase of creating questions under the
Making questions as the way favourable for students’ mathematical activity

The described study is a part of a larger project. It is aimed at checking and comparing how the proposed method increases the mathematical activity of students. Previously, the similar lesson scenario has been implemented with students of the first class of high school, where mathematics is tough at the extended level and with third-grade secondary school students. Based on the interview conducted after completing the research in these three groups, the following conclusions can be posed:

1. The necessity to create questions induced respondents to return to the given definitions in order to complete the task, contributing with a better understanding this theory.
2. The formula of the exercise encouraged systematization, ordering and creative application and processing of mathematical information.
3. Exercises with questions provoked students to submit their reasoning under the assessment and allowed for a “look back”, resulted with optimization and reasoning of ordering.
4. The participants have made an attempt to create a set of the most universal questions that can be asked on the path to solve further problems.
5. The application of knowledge and skills in a non-standard situation encouraged the respondents to increase their mathematical activity.
6. Making questions provoked the participants to pay more attention to the language, concepts, definitions, relationships between objects.

Making questions, parallel to solving the basic, mathematical problem, is to be a response to the “[attention] paid by A. Z. Krygowska to the problems related to the development of attitudes and methods typical for creative mathematical activity”. What’s more, this method is available for every student, regardless of their experience, opportunities, level of interest in mathematics.

BIBLIOGRAPHY.

Skurzyński K. (1963), Zapamiętywanie mimowolne w procesie nauczania matematyki. Matematyka. Czasopismo dla nauczycieli, nr 3(77), Warszawa, s. 90-95.
STAWIANIE PYTAŃ JAKO METODA SPRZYJAJĄCA
MATEMATYCZNEJ AKTYWNOŚCI UCZNIÓW

ABSTRAKT

„Pytać dobrze – znaczy uczyć dobrze” (C. De Garmo, 1911) to słowa, które stały się inspiracją do prób wykorzystania pytań zadawanych przez nauczyiciela do zbudowania metody, która pozwoliłaby uczniowi rozwijać swoje umiejętności i pogłębiać wiedzę. Wszak pytania, nieuniknione w przestrzeni szkolnej, nie muszą służyć jedynie sprawdzeniu stanu wiedzy ucznia. Pytania nie muszą również służyć jedynie uzyskiwaniu jednej, ocenianej odpowiedzi. Pytania nie muszą wreszcie pełnić jedynie roli jednostronnej formy komunikacji.

Na podstawie dotychczasowych badań, metoda stawiania pytań zdaje się sprzyjać zwiększeniu matematycznej aktywności uczniów. Ci, którzy wykazali większą aktywność i zaangażowanie na polu tworzenia i opracowywania postawionych przez siebie pytań, finalnie rozwiązywali postawiony problem znajdując poprawną odpowiedź. Mówią o tym sami badani, którzy w rozmowach przeprowadzonych po zakończeniu eksperymentu przyznają, że ćwiczenie polegające na postawieniu serii pytań, a następnie na pracy z nimi, pozwoliło im na powrót do zagadnienia, powtórne przemyślenie drogi jego rozwiązania, zastanowienie się nad optymalizacją tego procesu.

W swoim artykule proponuję – sprzyjającą pobudzaniu matematycznej aktywności uczniów – metodę pracy na lekcji matematyki.

Słowa kluczowe: kognitywistyka, edukacja matematyczna, poziomy przetwarzania informacji, nowe media